Prove $\dfrac{\cot C - \tan C}{\cos C + \sin C} = \dfrac{\cos C - \sin C}{\cos C \sin C}$.

$$\downarrow$$

$$= \frac{\dfrac{\cos C}{\sin C} - \dfrac{\sin C}{\cos C}}{\cos C + \sin C} \cdot \frac{\sin C \cos C}{\sin C \cos C}$$

$$= \frac{\cos^2 C - \sin^2 C}{(\cos C + \sin C) \sin C \cos C}$$

$$= \frac{(\cos C - \sin C)\cancel{(\cos C + \sin C)}}{\cancel{(\cos C + \sin C)} \sin C \cos C}$$

$$= \frac{\cos C - \sin C}{\sin C \cos C}$$

$$\frac{\cos\theta}{\tan\theta+\sec\theta}$$

$$= \frac{\cos\theta}{\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}} \cdot \frac{\cos\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta+1}$$

$$= \frac{1-\sin^2\theta}{\sin\theta+1}$$

$$= \frac{(1-\sin\theta)\cancel{(1+\sin\theta)}}{\cancel{\sin\theta+1}}$$

$$= 1-\sin\theta$$

$$\frac{1}{(1-\sec^2\beta)\cos\beta}$$

$$= \frac{1}{-\tan^2\beta\cos\beta}$$

$$= \frac{1}{-\frac{\sin^2\beta}{\cos^{\cancel{2}}\beta}\cancel{\cos\beta}}$$

$$= -\frac{\cos\beta}{\sin^2\beta}$$

$$= -\frac{\cos\beta}{\sin\beta}\,\frac{1}{\sin\beta}$$

$$= -\cot\beta\csc\beta$$

$$\frac{\cos x}{1-\sin x}-\frac{1-\sin x}{\cos x}=2\tan x$$

$$\downarrow$$

$$= \frac{\cos^2 x-(1-\sin x)^2}{(1-\sin x)\cos x}$$

$$= \frac{\cos^2 x-(1-2\sin x+\sin^2 x)}{(1-\sin x)\cos x}$$

$$= \frac{\cancel{1}-\sin^2 x-\cancel{1}+2\sin x-\sin^2 x}{(1-\sin x)\cos x}$$

$$= \frac{2\sin x-2\sin^2 x}{(1-\sin x)\cos x}$$

$$= \frac{2\sin x\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}\cos x}$$

$$= 2\tan x$$

$$\csc^2 A + \sec^2 A = (\cot A + \tan A)^2$$

$$= \cot^2 A + 2\cot A \tan A + \tan^2 A$$

$$= \csc^2 A - \cancel{1} + \cancel{2}\frac{1}{\cancel{\tan} A}\cancel{\tan} A + \sec^2 A - \cancel{1}$$

$$= \csc^2 A + \sec^2 A$$